# FORMALIZATION AND INTUITION IN *ANALOGIQUE A ET B*

**(with some remarks on the historical-mathematical sources of Xenakis)**[1]

Agostino Di Scipio

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**ABSTRACT**

*Analogique A et B* (for 9 string instruments and tape, 1958-59) is considered one of Xenakis' most thoroughly formalized compositions. Two chapters of *Musiques Formelles* are dedicated to theoretical and technical issues behind it, mostly dealt with by Xenakis using mathematical notations and other formalisms. However, in this paper I argue that a very rich exchange took place, in the composition of this music, between formalization and more intuitive insight.

The paper first overviews the mathematical sources of Xenakis' efforts in the late 1950s, and then illustrates details of the "mechanism" (formalized compositional process) by which he created *Analogique A* (the instrumental layer of *Analogique A et B*). The composer pursued a generative process that would function as the *analogon* of a stochastic process, the latter consisting in what Xenakis called *sound clouds*. A close look at the procedures he implemented to that aim, shows that (1) a binary logics informs both the "mechanism" and the final musical results, and (2) the composer actually created a discrete, *digital* representation of the stochastic process he had in mind. The paper discusses the dialectic between formalized and intuitive choices, including Xenakis' final decision to paste together the two separate manifestations of the same design, the instrumental (*Analogique A*) and the electronic (*Analogique B*).

«Le nombre n'a donc pas éliminé l'inconscient. Mais quel est son rapport au sens, et à l'inconscient, le nombre ne peut pas le dire» (Henri Meschonnic [20: 576])

## 1. PREMISES

*Analogique A et B* (1958-59) is one of the compositions whose theoretical premises Xenakis discussed at length in the book *Musiques Formelles* [30]. Chapters 2 and 3 of the book are devoted to theoretical and technical issues having a role in that work. In those Chapters, Xenakis proposed the theory of Markovian Stochastic Music, and illustrated its application in two separate but essentially similar pieces, namely *Analogique A* (for 9 string instruments) and *Analogique B* (defined as "electromagnetic music" for "sinusoidal sounds", recorded on tape). In the same circumstance, Xenakis introduced a representation of acoustical signals of a kind first proposed by Dennis Gabor in the 1940s, where finite base functions replace the infinite functions usually adopted in more widely accepted representations (Jean-Baptiste Fourier's, dating from 1807 and 1822). With his quantum-oriented representation of sound (that he later preferred to date back to earlier propositions by Albert Einstein, not to Gabor), Xenakis could

[1] The present paper is the revision of a draft initally prepared for a lecture (April 27, 2002, at IRCAM, Paris, in a meeting on Music and Mathematics organized by Moreno Andratta and Stephan Schaub), and is based on previous work as yet unpublished [7].

arrange innumerable acoustical quanta in time and create “clouds of sounds” (his term). This is today recognized as the first effort in *granular* synthesis of sound ever made in musical contexts, albeit a technologically problematic and relatively efficient effort. Together with the tape work *Concret PH* (1958), realized with a rough technique of granular *transformation* (not *synthesis*) of sound, *Analogique B* represents the first music ever made leaning on a corpuscular view of the physical world (a view echoing Gassendi’s corpuscular mechanicism, and particularly some hypotheses concerning the atom-like nature of sound proposed by Isaac Beekman, in the 17th century).[2]

In the following, I’d like to focus on the compositional “mechanism” designed and utilized in the making of *Analogique A et B*.[3] My discussion will refer particularly to the composition of the instrumental piece, *Analogique A*. However, on a more general level of discourse, the goal is to provide elements for an informed discussion concerning the duality (proper to all music composing, in my opinion, albeit in different shapes and degrees) between *formalizable* and *non-formalizable* aspects of the creative process of composing.

I believe we need to be as deeply aware as possible of the mathematics and other conceptual tools Xenakis set up for himself, in order to also be able to catch on more intuitive aspects that are indeed crucial to his music. We should bring ourselves to the border beyond which compositional decisions and choices are found that evidently could not be dealt with in a systematic and wholly rationalized approach, and that were dealt with by the composer in more qualitative and informal – maybe unformalizable – manners. Only by that border can we try to somehow characterise the necessary interactions taking place between the two domains, the outer and the inner. Such an approach is particularly appropriate, I believe, when dealing with composers, like Xenakis, who are or were not just a *users* but also *designers* of their own working tools, composers of the music *and* of the technologies involved. The knowledge Xenakis put into the making of his music is, in most cases, not at all secondary to the music itself.

I must also say that *Analogique A et B* is often regarded as a very problematic composition. Some would say it is one of the least successful works ever composed by Xenakis.[4] That is usually explained with the strong emphasis he put on the theoretical and technical details, summing up to an overload of formalistic premises whose result seems to be musically rather poor. That opinion is also reinforced by a misunderstanding of Xenakis own dissatisfaction with the results he could achieve - which had primarily to do with the technological limitations he experienced in the realization. In short, for many *Analogique A et B* is little more than an unsatisfactory experiment. In a way, that is absolutely correct. Evidence being that Xenakis never took up the particular approach again in later works (in my opinion, though, such circumstance tells us little).

In contrast with that view, though, I would say that the problematic aspects of *Analogique A et B* ultimately represent the very element giving this music a strongly peculiar, almost unique character, also quite palpable when listening to it. Some of the problems Xenakis raised in composing this work, and that he apparently left without satisfying solutions, resulted into choices and decisions that remained (and probably had to remain) non-formalized. This music is not the audible trace of a thoroughly formalistic approach, but the result of a clash between different domains of rationality. It entailed not only a substantial body of theoretical premises, but also important manual, non-formalized adjustments and arbitrary choices. It represents less an unsatisfactory experiment than a work expressive of a

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[2] For more on this topic, see [6] and various passages of [23].

[3] “Mechanism” is Xenakis’ own term, found in his discussion of Markovian Stochastic Music. According to Sharon Kanach [13], before calling his book *Musiques Formelles*, Xenakis had considered the title *Mecanisme d’une musique*.

[4] e.g., Solomos writes «Le résultat n’est pas convaincant...» [27:35]. It must be noted, on this point, that Hermann Scherchen disliked this work [17: 135]. Harley suggests [11: 24] that Xenakis in 1959 wrote *Syrmos*, dedicated to Scherchen, in an attempt to create a more convincing music based on the same grounds as *Analogique A*. Harley adds that *Syrmos* is «much more engaging to listen to» [ibid.].

lively and intricate dialectic between formalization and intuition. Interesting is not simply the opposition *formalization* vs. *intuition*, *machine* vs. *human*, but the dialectic between such terms in the actual process of knowledge that we call composing. The two elements are inextricably intertwined, so interlaced that they can hardly be separated in actual experience (neither the composer's, nor the listener's).

On one hand, Xenakis' work certainly owes to a formalistic modern tradition of thinking. In that tradition – going from Descartes, Kant and Leibniz down to Russel, Turing, von Neumann and Wiener – form and content, symbols and objects are separated and independently discussed. On the other hand, at some point Xenakis must anyway rejoin form and content. This, in theory, he did *a priori* via axiomatisation – again an element of formalistic tradition. But he did it, too – empirically and often *a posteriori* – based on experience and non-formalizable (or not-yet-formalized) insight. *Where* and *how* form and content are rejoined and become no longer separable? That is the object of our study here.

Were the above premises to be proven correct, then *Analogique A et B* should be regarded not at all as a failure but as a masterwork: in a very subtle way, it makes the dialectic between opposite attitudes of human knowledge (as admittedly all too simplistically captured in the opposition of formalization vs. intuition) *the* issue at stake in this music.

Before delving into the details of Xenakis' constructive mechanism, it is useful to briefly consider some mathematical sources found in Xenakis' writings on Stochastic Music, and related general issues in the history of science that may be relevant to better understand the conceptual context to his compositional "mechanism".

## 2. SOURCES OF THINKING

### 2.1 Direct and Indirect Sources

In Xenakis' early musical and theoretical efforts, several echoes are found of his interest for mathematics and physics.[5] Certain theoretical issues recur quite often – consider, e.g., his necessity to reshape the *foundations of music*, reminiscent of the quest for the *foundations of mathematics* that had been crucial in late 19th-century science and that still reverberated at the time when Xenakis became involved in composition. Consider his interest for the question of *determinism* and *indeterminism*, and for the question of the *continuum* (of numbers, of time…), whose audible musical trace – the *glissando* – eventually became almost like a trademark of his music.[6] And consider, in a more specific example, the emphasis Xenakis put on the Theory of Probabilities and specifically on *continuous probability functions*, that he turned into numerical processes of use in shaping meaningful sonic structures.

Many times Xenakis evoked the names of important figures in the history of science. Most of his sources in the field should probably be drawn back to the early decades of the 20th century, namely to scientists and researchers who, in turn, had dealt with important questions first asked in the 19th century. Xenakis was clearly aware of the scientific revolution that had taken place in the early years of the 20th century in physics. Thermodynamics, a science often mentioned in his earlier work, was a branch of physics of major importance in the natural sciences of the 20th century, and was decisive for the development of Information Theory (and the Theory of Communication, as it came to be called), with enormous repercussions on modern technologies. Information Theory was becoming of age and of vast social impact precisely at the time when Xenakis started composing.

[5] Solomos [28: 128 and 131] observes that Xenakis had a preference for metaphors drawn from physics and natural sciences, as opposed to more abstract mathematical thinking.

[6] Xenakis touched upon the question of the continuum already in the first Chapter of *Musiques Formelles*, as he introduced Free Stochastic Music [30: 9]. (In the present paper, page numbers for [30] refer to the English translation).

Xenakis' work from the late 1950s and early 1960s shows a familiarity with the principles of then-young areas of scientific concern, crossing the boundaries of many apparantly distant disciplines. I refer to General System Theory and Cybernetics. He had a special admiration for Jean Piaget, too. References to Piaget are found both in one of Xenakis' earliest introduction of Free Stochastic Music [30: 5], and in one of his latest papers [34]. Although rooted in a different tradition, Piaget's work not by chance became an important source for thinkers and scientists active in the so-called 2nd-order Cybernetics and fostering modern constructivistic epistemologies (a.o. von Foerster, Ashby). (All such authors share both an apology of the systemic meaning of "chance" or "noise", and a dissatisfaction with the neodarwinian role of aleatoric processes in the creative social and cultural context. Xenasis, apologetic of indeterminism and yet appalled by the Cagean *alea* and more in general by improvisation, would have agreed with that.)

Xenakis' interest in these matters certainly stemmed from his own personal attitude and genuine curiosity for all science (and philosophy), but it was also increased and reinforced by the fact that, at the time of his connection with the GRM in Paris (late 1950s), he met with Abraham Moles, whom surely he owed familiarity with several publications in Information Theory and Cybernetics.[7] Xenakis read of Dennis Gabor's quantum approach on sound and hearing [10] in Werner Meyer-Eppler's writings on Information Theory [19], or heard about it already in lectures that Meyer-Eppler had delivered earlier in Gravesano (as is known, most of the writings eventually collected under the title *Musiques Formelles* had first been published in the review Gravesaner Blätter, upon the initiative of Xenakis' mentor, Hermann Scherchen).[8]

Another direct source, on related issues, was the work of the French mathematician Maurice Fréchet (1878-1973) – who had worked on the Theory of Probability since the late 1920s, and whose main publication in the field Xenakis knew [30: 79].[9] Fréchet's book was also mentioned in the then-recent ground-breaking contribution by Shannon & Weaver, *The Mathematical Theory of Communication* [23: 49 and 51]. Xenakis approached Shannon's book with an awareness that Information Theory was grounded in concepts of thermodynamics [30: 61].[10] In making a link

[7] In Moles' 1958 book we find a distinction between "micro- ", "meso-" and "macro-structure" [21: 184], one that was probably of special relevance to Xenakis. (Page numbers for [21] are referred to the Italian translation). – Or was it rather Moles owing Xenakis a hint to such a distinction? In his 1960 discussion of Markovian Stochastic Music, Xenakis touches on the question of the "scales of observation" that would be pertinent to the study of sound phenomena, and distinguishes between "microsounds" and the "macroscopic methods" (statistical procedures) necessary to handle sound particles and complex sounds [30: 49-50]. However, to be honest to Moles, we should say that only much later Xenakis replicated the three-fold distinction "micro- ", "meso-" and "macro-structure" [30: 266].

[8] According to Solomos [29: 7], Xenakis could attended a lecture given by Meyer-Eppler (*Metamorphose des Klangelemente*) in Gravesano as early as 1955. The hypothesis is reasonable and, if it is correct, then Xenakis would have known of the music-related research Meyer-Eppler was pursuing at that time. Meyer-Eppler also presented his research in a paper included in the first issue of the review *Die Rehie* (published 1955 in German, and then translated in English in 1958) [18]. Indeed, the second section of that paper has materials that could have been crucial to Xenakis.

[9] The subtitle of Fréchet's book [8] is worth mentioning here: *Theorie des événements en chaîne dans le cas d'un nombre fini d'états possibles* (Theory of Event Chains...), a direct reference to Markovian processes. It must be noted that Fréchet had been one of the most brilliant pupils of a greatest French mathematician, Jacques Hadamard (1865-1963). Hadamard is known, among many other things, for the so-called *Hadamard transform* (or *Walsh-Hadamard* transform), that is a generalized class of Fourier transforms based on squared – not sine – functions. The Hadamard transform never really raised any interest in acoustics and signal processing, but it has been of use for a *quantum* approach on information processing.

[10] The date of Fréchet's book [8], as reported in *Musiques Formelles*, is 1952, but the book seems to have actually appeared in 1938. Probably Xenakis was referring to a later reprint, or maybe just failed to correctly report the bibliographical details. Shannon & Weaver correctly dated Fréchet's book from 1938 [25: 49]. It is worth observing that in Fréchet's book many issues were discussed of the highest interest for Xenakis, including the Birkoff *ergodic* theorem. The definition of "ergodic process" is also found in Shannon & Weaver (discussed next to a discussion of Markov chains [25: 50]), and of course is found in *Musiques Formelles* [30: 56 and 67], where the reference is in fact Fréchet, not Shannon. Xenakis himself has noted later: «I took the definition [of ergodic process] from the book of that important French mathematician, Maurice Fréchet, who has

between Boltzmann and Shannon [30: 61 and 255], he was absolutely correct: the intersection of Probability, Thermodynamics and electrical Communications «is made in the moment when Boltzmann equals *entropy* with *probability*, and in the moment when Shannon equals *entropy* with *information*» [3: 787, my translation].[11]

Xenakis envisioned the use of computers as a way to put his efforts into practice, already in his very first discussion of Free Stochastic Music (that was based on previous compositional experience, as in *Pithoprakta*, 1955-56, and in *Achorripsis*, 1956-57). As is known, he eventually accessed computers only in 1962, when he implemented the ST program on an IBM7090 mainframe computer. That opened to what was later called "algorithmic composition", and indirectly to "non-standard methods of sound synthesis" too, that Xenakis himself developed later.[12] Like very few composers of his generation (including a.o. Gottfried M. Koenig, Herbert Brün and the Italian Pietro Grossi), he learned some computer programming. Of course his program codes – either the ST program (written in Fortran language, 1962, [30: 145-152]) or the first version of the GENDYN program (written in Basic, early 1990s [30: 300-321]) – are certainly far from any professional computer programming standard. Still, they reflect a musical vision that, at the time, challenged the very concept and meaning of composition. His analysis of the «general phases of a musical work» [30: 22] should be seen as the first example of *composition-theory*, to use a term first coined by Koenig and later reprised by Laske (e.g. [15]). Already in 1962 Xenakis felt that, in a fully automated art, the computer program would represent «the objectification of musical form» [30: 29][13], a kind of utopia Xenakis could get a little closer to only some 30 years later (with compositions such as *Gendy3*, 1991, and *S.709*, 1994).

### 2.2 Mathematical Inclinations

In Xenakis' early writings we often find mathematical notations borrowed from the work of 19$^{th}$-century scientists – names such as Poisson, Cauchy, Gauss, Boltzmann (kinetic theory of gas), Bernoulli (inventor of the term "stochastic system") and many others come to mind. Augustin-Louis Cauchy (1789-1857) and Carl Fredrich Gauss (1777-1855), in particular, were not only important mathematicians of their time, but also participated in the early debates on the foundations of mathematics. They held two very different views. Cauchy fostered what could be called an "ontological" approach – he asked *what is a number? what is a mathematical object or entity?* Gauss was more methodologically inclined, as for him the question was rather *how numbers function? what are the limits in the application of mathematical methods?* In the first view, fundamental is the very concept of number, or anyway some thing that could be called a numerical entity, an object having a numerical nature. In the second, fundamental is the procedure by which

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written on Markov chains» [33: 53]. All these circumstances suggest that Xenakis came to know first Fréchet's book and then Shannon's (the former he came to know by himself, the latter through Moles), and that Shannon's book must have been for him like a source that confirmed issues he was already familiar with. This conjecture is consistent with the fact that in Moles' book no mention is found of Fréchet nor of any other technically and mathematically-oriented reference on Information Theory *except* Shannon & Weaver.

[11] In a later inteview [24: 17], Xenakis touches again on Shannon & Weaver's book, but only to comment, in some detail, on the possibility to build up a language based on the free combination of symbols belonging to a set, given the probabilities for these symbols to occur in a sequence (a task in statistical linguistics). A significant similarity can be noted between the way Shannon treats his "discrete source of information" [25: 42] generating strings of symbols (letters and words) and the way Xenakis discusses his "mechanism", illustrating its output data as strings of letters ("protocols"). [30: 97-98].

[12] In this context, "non-standard sound synthesis" means an approach to the computer-based generation of sound using models and theories having little or no pedigree at all in academic areas such as acoustics and signal processing. Examples include Koenig's SSP program, Paul Berg's PILE program, Brün's SAWDUST project, and this writer's FIS (a method of sound synthesis based on the iteration of nonlinear functions). Xenakis' Dynamic Stochastic Synthesis (as first utilized in passages of *La Légende d'Eer*, early 1970s, and then in his GENDYN program, early 1990s) must be included in this list.

[13] Page numbers for [32] are referred to the Italian translation. All quotations from [32] are my own English translation.

objects can, regardless of their truth or essence, be mathematically operated upon. At the time, similar questions were raised when discussing such things as the continuity of a function, the notion of infinite and infinitesimal quantities, and other general issues in analysis and calculus. The divergence between Gauss and Cauchy is nicely captured in the Gauss' observation that the infinite can only be postulated: the infinite, he claimed, is only *une façon de parler* (a manner of speaking).[14]

Interestingly, Gauss mantained that the theory of numbers could be only grounded on arithmetical methods capable of illustrating theorems and capable of being linked to other theorems, therefore forming a chain of theorems (in fact a *theory*). The term "arithmetical method" means, in this context, a demonstrative procedure that link from one theorem (one elementary concept or observation in the theory) to the next, thus forming a concatenation of separate theorems. A method was for Gauss a «demonstrative process having a *discrete* rhythm… allowing to shift from one entity to the following one or to the preceding one» [1: 88, my translation]. For him, it was wrong to assume a numerical *nature* of anything – it is rather the reliability of humanly-devised arithmetics which, in a way, determines the boundaries within which numbers can be said to "exist". More in general, Gauss didn't accept the theory of the *continuum*, he rejected the Platonic and continuist view of Bolzano, Cauchy and others – although certainly in his work he did utilize calculus and continuous functions. Gauss shared Galileo's opinion that, while we can indeed speak of "atoms", we cannot measure and assign them any quantity: *atomi non quanti*. He was against the hypothesis of the reality of infinite and infinitesimal quantities, and used to call these "potential quantites" (as opposed to "actual" ones). Gauss would claim that the continuous and the discrete, the infinite and the finite can not and should not be treated by the same methods, as there is a leap between them. For him, scientific rigour and precise formalization were a matter of wise methodological strategies, whose success could not be granted by the existence of numerical entities.

On the relationship between the continuous and the discrete, Cauchy held a different position. He is usually credited to have set the premises that make it possible to operate exchanges betweeen continuous and discrete domains, while also allowing a separate treatment of the continuous and the quantized – of the *analog* and the *digital*, as we say today. This was crucial to technical matters developed in the early 20th century, including Alan Reeves' description of a method for discrete measurements (sampling) of continuous signals later known as Pulse Code Modulation (PCM). (Reeves research dates from the 1930s, and – as is known – much later PCM became a technical prerequisite to almost all digital audio technology).

Xenakis did not put any particular emphasis on numbers as such, except perhaps for his early interest in the Fibonacci series – already in *Tripli Zyia* (1952) and the macrostructure of *Metastaseis* (1955).[15] We could say that, in a Gaussian perspective, Xenakis emphasized "methods", that is, the planning of well-defined procedures, the logics of ways of proceeding (μετα–οδος). He very clearly stated that Stochastic Music was a general "method", not to be confused with a personal style.

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[14] For more on these different lines of thought in the 19th-century foundations of mathematics, see [1].

[15] As also noted by Orcalli [22: 36], Xenakis probably borrowed an interest for the Fibonacci series from Le Corbusier. At the time, Xenakis was still an assitant to the famous architect, and knew very well Le Corbusier's Modulor [31]. Anyway, the Fibonacci series must have been familiar to several of Oliver Messiean's students, including Karlheinz Stockhausen (e.g., *Kreuzspiel*, 1951). Stockhausen himself has later claimed he had already used Markov chains in his work in the mid 1950s, «highly influenced by my teacher, Meyer-Eppler» (he defined Shannon «an important mathematician» and «Markoff, too») [2: 67-68]. Although the German composer never explained exactly how «he trasposed everything I learned [on this subject] into the field of music» [ibid.], what he did was to create an abstract language starting with a set of given syllables (or even phonemes), and then replicating Shannon's use of Markovian processes [2: 67] (see footnote 11).

On the other hand, in a way closer to Cauchy, the question of the continuum is central to some of Xenakis' theoretical speculations, as well as to some of his compositions. The question of the continuum is also to be connected with the name of George Cantor (1845-1918), whose propositions had a strong impact in scientific and epistemological circles in the late 19th century, and were very hotly debated and even rejected by many.[16] It is very likely that Cantor's Theory of Infinite Sets, first explained by the mathematician in letters to friends written in a rather informal and very passionate style, was particularly dear to Xenakis.[17] In a late paper [34], Xenakis mentioned the notion of "ordered structure" in terms that seem to have been taken directly from Cantor (he made no explicit reference, though).[18] It must have been clear to him that Cantor's contributions shed some light over a number of fundamental antinomies and inconsistencies in classical mathematics, thus raising critical questions discussed at length in the late 19th century, and still reverberating well into the 20th century.

Although he often discussed and utilized continuous probability functions, Xenakis was never explicit on the issue of the necessary quantization he had to operate when mapping from the continuum to discrete spaces (e.g. musical pitch, rhythm pattern, etc). Probably he took the issue for granted, as an obvious point that goes without saying. We should observe, though, that it is precisely here, in the mapping operation, that Xenakis could adjust or anyway change the results of his calculations. However utterly formalized or mathematized his music might appear to us, it is clear that no formalization of mapping is found in his writings. That is little surprise, in a way, as mapping must always follow non-generalisable strategies, depending on practical circumstances (instrumentation, type of notation, etc.) and/or particular musical goals, and hence it always requires *ad-hoc* solutions.

We can turn, now, to an overview of the compositional "mechanism" behind *Analogique A*. Both in the technical details and in the discussion, some issues will reverberate from questions shortly touched upon above. For lack of space and sake of clarity, I must crudely simplify some technical points. However, that should not be too big a problem, here, as the focus of the analysis will be more on the overall function and qualitative meaning of the compositional process. For details on aspects too briefly discussed here, please refer to [4] [5] and [6].

## 3. COMPOSITIONAL MECHANISM OF *ANALOGIQUE A*

### 3.1 Variables

To start with, Xenakis makes the decision that his "compositional mechanism" will operate upon three variables: pitch, dynamics, and density.

*3.1.1 Pitch*

"Pitch" means here (in *Analogique A*) discrete pitch as from the traditional equal-temperement system (in *Analogique B*, it means instead "frequency", which of course is not the same thing). The range of available pitches is divided into six smaller ranges or regions:

[16] Henle [12] includes Cauchy and Cantor in a line of "Romantic mathematics", as distinct from other ages of mathematical thought (Renaissence, Baroque, Classical and Atonal mathematics). Let's recall that, among the scientific theories taken on by Xenakis, Maxwell's Kinetic Theory of Gases is from 1860-65, that the discovery of Brownian motion is from around 1867, and that the first notion of entropy was provided by Clausius in 1865.

[17] In an interview from the mid 1980s, Xenakis observes: «Il n'y a rien qui puisse s'inventer, surtout dans les sciences, sans une vision très au-delà des conventions et des outils techniques. Un homme comme Cantor, l'inventeur de la théorie des ensembles, a lutté toute sa vie, grâce à une impulsion interne, grâce à un rêve très riche» [16: 224].

[18] Xenakis: «Given three elements of a set, they can be ordered in just one way by saying that one of the three is between the other two. In other words a set of elements has an ordered structure if you can put them in a string, placing each between two others...» [34: 144]. For Cantor, a set is "ordered" if, given «any two elements *a* and *b*, either *a* precedes *b* or *b* precedes *a*; moreover, if *a* precedes *b* and *b* precedes *c*, than *a* precedes *c*» (quoted in [14: 229]).

I = $E_0 \dots E_1$
II = $E_1 \dots D_2$
III = $D_2 \dots Db_3$
IV = $Db_3 \dots C_4$
V = $C_4 \dots B_4$
VI = $C_4 \dots A_5$

The regions are grouped into two sets, that Xenakis calls $f_0$ and $f_1$. The first set comprises regions I, II, V and VI. The second comprises the two remaining regions:

$f_0$ [I, II, V, VI]
$f_1$ [III, IV]

The two sets are prefectly complementary : their sum equals the complete set, comprising a single instance of each component region. (It should be observed, incidentally, that Xenakis did not seem to have clarified whether the two extremes in a particular set or region do or do not belong to the set. For example, does $D_2$ belong to region II or III? Does it belong to both? In case of a positive answer to the latter question, one should then admit that $D_2$ belongs to both sets $f_0$ and $f_1$).

When set to work, the compositional mechanism will, at each next step, (1) select one of the two pitch sets, (2) select a region from within the selected set, and finally (3) pick a pitch from within the selected region. The set selection follows specific rules, which will be the object of further analysis below. Differently, the region and the individual pitch within the region are selected on a purely random basis – that is, using a flat probability function (all chances equally weighted).

*3.1.2 Dynamics*

Three values are considered:

I = *pp*
II = *f*
III = *fff*

These are grouped in two sets:

$g_0$ [I, I, II, III]
$g_1$ [I, II]

The first set comprises all three different values, but includes two instances of the value *pp*. In other words, four items are included, one of the three values being assigned twice as many chances as the others. The second set, $g_1$, comprises instead two values, with equally wighted chances. The compositional mechanism, once running, will (1) select one of the two sets, and (2) pick a value from those available in that set. Again, the set selection has its own rules, while the latter is made on a purely random basis.

*3.1.3 Density*

Density means here "average amount of events in the time unit". Three values are considered:

I = 1 event / $\Delta t$
II = 3 events / $\Delta t$
III = 9 events / $\Delta t$

where "events" mean "notes" (more precisely, "note onsets"). The time unit $\Delta t$ equals a half-bar duration. And with ♩ = 50 (all throughout the score to *Analogique A*), that duration is 1.2".

Density values are grouped in two sets:

$d_0$ [I, I, II, III]
$d_1$ [I, II, II, III]

Set $d_0$ comprises all three values, but has two instances of value I. Set $d_1$ itself comprises all three values, but has two instances of value II.

### 3.2 Time grid

Note durations are not considered as variables to be handled by the compositional mechanism. They are freely assigned by Xenakis according to another strategy, indeed a very simple one. As is clear from the score, three instruments always play quintuplet notes against the half-bar duration, three other instruments always play quadruplets, and the remaining three always play triplets. Note durations always equal a single subdivision of the rhythm group to which they belong. Xenakis thus arranged for a fixed time grid where notes should be appointed, depending on given rules (a similar arrangement is often found in Xenakis' pieces from the late 1950s).

Accordingly, there are 5 + 4 + 3 = 12 time positions in Δt, constituting a discrete time grid, a fixed quantization of the time continuum across the entire musical piece. And there are, too, 3 × 12 = 36 locations or places Δt, to which the sounds output by the compositional mechanism can be assigned. However, as Xenakis fixed a maximum amount of events of either 14 (when density set $d_0$ is selected) or 16 (when $d_1$ is selected), in actuality only less than 50% of the 36 available locations are appointed a note. Moreover, different from what he did about pitch and dynamics, Xenakis managed density rather freely, seldomly using the very values selected by his own mechanism. Typically, the amount of events in Δt is much lesser than 50% of the available locations on the time grid.

It is worth noticing that density values were arranged by Xenakis logarithmically (1, 3, 9 events). That appears consistent with the Fechner law, in a perceptual domain ("density") that at the time had not been yet investigated (and probably not even recognized as such) by psychoacousticians.[19] Today we know that the Fechner law does not hold below and above some thresholds, not even with regards to the seemingly most obvious dimensions of musical perception – pitch and duration. However, Xenakis was probably right to make such a decision concerning density, especially to the extent that the particular values involved are neither too small nor too big.

### 3.3 Method

Xenakis governed the set selection with Transition Probability Matrices (TPM). An example matrix discussed by the composer (and one that he did actually use in composing *Analogique A et B*) is:

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[19] According to the founder of psychophysics, Theodor Gustav Fechner (1801-1887), magnitudes in sensation follow the logarithm of magnitudes in the stimuli: perception of constant linear increments reflects objective constant ratios. In the domain of musical perception, the typical example is the perception of pitch: pitch distances (intervals) are perceived as linear increments but are actually measured by logarithmic increases in frequency. It is interesting to note that one of Xenakis' most brilliant exegetes, philosopher Michel Serres, in a Chapter called *Mathématisation de l'empirisme* addressed the similarity between the Fechner's law for sensation ($S$) and the definition of information ($I$) as in Information Theory:

$S = k \log I$ (where $I$ is a magnitude in the stimuli)

$I = k \log P$ (where $P$ is a magnitude in the probability that some event occurs).

Serres writes: «La notion d'information est utilisée en physique et théorie de la communication de manière indépendent du sens du message qui la transporte» [26: 195]. And also: «La notion de sensation est utilisée en psychologie de manière indépendante ... du stimulus qui la produit» [ibid.]. We have, here, two historically specific examples a formalistic tradition of thinking. Leaning on that tradition, Serres also writes «Dans le deux cas, la mathématisation est possible à la condition exclusive de mettre en parnethèses la question du sens» [26: 196], and therefore he points out a contradiction: «information et sensation sont des grandeurs absolues ... *formalisables* et *informelles*» [ibid., my emphasis].

|  | $f_0$ | $f_1$ |
|---|---|---|
| $f_0$ | 0.85 | 0.4 |
| $f_1$ | 0.15 | 0.6 |

It means that, starting with the pitch set $f_0$, there will be 85% of chances that again $f_0$ will come next, and 15% chances that $f_1$ will come next, instead. Starting with $f_1$, on the other hand, there will be 60% chances that $f_1$ again will follow, and 40% chances that, instead, $f_0$ will follow. This represents a so-called Markov chain, or Markovian process.[20] The value selected next, is statistically dependent on the value currently selected.

The stochastic process thus described is different from strategies Xenakis had been operating in previous works of Free (not Markovian) Stochastic Music. In *Pithoprakta* and *Achorripsis* the event distribution had been deliberately based on «an aleatory law *without memory*» [30: 23, my emphasis]. By leaning on a 1st-order Markovian process (a process with a single memory cell), the composer instead pursued a probabilistic mechanism operating *in time*, not abstract from time. The model, in Xenakis' view, would work in analogy to a process unfolding in time, apparently random in its behaviour, but having significant (and possibly audible) structural properties. In a way, the compositional mechanism of *Analogique A et B* is probably the first *in-temps* structure Xenakis ever formalized. With *Analogique B*, the very same *in-temps* structure was to be projected to a micro-temporal scale.

We know that, after *Analogique A et B*, Xenakis has never taken up the approach of Markovian Stochastic Music again. It seems reasonable to argue that, already at a time when he had not yet introduced the famous *in-temps / hors-temps* distinction, he had a preference for *hors-temps* structures. In principle, the genesis of that distinction could be discussed based on a comparison between the constructive methods of *Achorripsis* – where separate, independent stochastic projections were utilized, not making up a process as such – and *Analogique A et B* – where, by using a chain, a process with a memory albeit minimal, he did point to let a true process unfolding in time.[21]

For each of the three musical variables, Xenakis utilized two TPMs to drive the set selection.

α

|  | $f_0$ | $f_1$ |
|---|---|---|
| $f_0$ | 0.2 | 0.8 |
| $f_1$ | 0.8 | 0.2 |

β

|  | $f_0$ | $f_1$ |
|---|---|---|
| $f_0$ | 0.85 | 0.4 |
| $f_1$ | 0.15 | 0.6 |

By using alternatively α and β, for the same amount of times, we achieve a more composite selection process. Applying the "probability-composition" rule, we find out the following weights:

$$p\,(f_0 \rightarrow f_0) = (0.2+0.85)/2 = 0.525$$
$$p\,(f_0 \rightarrow f_1) = (0.8+0.15)/2 = 0.475$$
$$p\,(f_1 \rightarrow f_0) = (0.8+0.4)/2 = 0.6$$
$$p\,(f_1 \rightarrow f_1) = (0.2+0.6)/2 = 0.4$$

We see here that the $f_0$ set is more likely to be picked up. In practice, it means that there are more chances that pitches will belong to extreme registers, those in fact grouped by Xenakis in the $f_0$ set. With a different grouping of pitch regions and sets, the musical result would most probably have been (very) different. In other words, the fact that the

[20] In 1962 Xenakis pointed out that the theory of Markov chains, first exposed in 1905 by the Russian mathematician Andrei Markov (1856-1922), was later developed in «parallel research carried out by Hostinsky, Potocek, Kolmogorov and Fréchet» [32: 29].

[21] The observation reinforces Orcalli's opinion that, already in a work like *Achorripsis*, Xenakis appeared more personally inclined towards *hors-temps* structures, towards the shaping of the overall «architecture of the compositional space» [22: 55].

most recurrent pitch collection comprises contrasting registers has little to do with the functioning of the mechanism in itself, and stems more from a decision made preliminary to that.

Surprisingly, the matrices used for the other two variables – dynamics and density – are identical with those used for pitch:

$$\begin{array}{c|cc} \gamma & g_0 & g_1 \\ \hline g_0 & 0.2 & 0.8 \\ g_1 & 0.8 & 0.2 \end{array} \qquad \begin{array}{c|cc} \varepsilon & g_0 & g_1 \\ \hline g_0 & 0.85 & 0.4 \\ g_1 & 0.15 & 0.6 \end{array}$$

$$\begin{array}{c|cc} \lambda & d_0 & d_1 \\ \hline d_0 & 0.2 & 0.8 \\ d_1 & 0.8 & 0.2 \end{array} \qquad \begin{array}{c|cc} \mu & d_0 & d_1 \\ \hline d_0 & 0.85 & 0.4 \\ d_1 & 0.15 & 0.6 \end{array}$$

Why Xenakis adopted three identical pairs of matrices is hard to tell. That way probably things were less cumbersome for him to handle. After all, he was implementing the whole process by hand, only using his pocket calculator. Computers were not yet available to him.

Next, Xenakis stipulated some *coupling rules* to connect the three pairs of matrices:

$$\begin{array}{cccccc} f_0 & f_1 & d_0 & d_1 & g_0 & g_1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \lambda & \mu & \alpha & \beta & \lambda & \mu \end{array}$$

$$\begin{array}{cccccc} f_0 & f_1 & d_0 & d_1 & g_0 & g_1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \gamma & \varepsilon & \gamma & \varepsilon & \beta & \alpha \end{array}$$

That means, for example, that

if pitch is selected from the $f_0$ set

then density value will be calculated with the $\lambda$ matrix,

and intensity value will be calculated with the $\gamma$ matrix

while

if pitch is selected from the $f_1$ set

then density value will be calculated with the $\mu$ matrix,

and intensity value will be calculated with the $\varepsilon$ matrix.

(etc)

It must be observed that, by means of these connections between TPMs of different musical variables, Xenakis actually introduced some constraints internal to the musical structure. That creates a psychoacoustically strong link among various dimensions of the musical texture. As an example, consider the second coupling rule: with set $f_1$ we have great chances that pitches from the middle registers are selected, and these will more often be followed by notes played *pp*, because that is the most likely value of dynamics to occur when matrix $\varepsilon$ (connected to $f_1$) is used. In other words, a correlation may be established among sound events, which may become relevant for the ear. Sure, it is a statistical correlation, i.e. one that is likely to reveal only as an average, actual occurrences being always slightly different. But it potentially introduces a perceptual redundancy in the music as heard.

Finally, observe that there are ultimately only $2^3 = 8$ combinations of sets available:

A = $f_0, g_0, d_0$ B = $f_0, g_0, d_1$ C = $f_0, g_1, d_0$ D= $f_0, g_1, d_1$

E = $f_1, g_0, d_0$ F = $f_1, g_0, d_1$ G = $f_1, g_1, d_0$ H = $f_1, g_1, d_1$

For these Xenakis used the word "screens". An illustration of the musical contents of such screens, is in Table 1 (see end of the paper).

The values obtained by the application of any of these screens constitue the statistical profile for the musical content of each half-bar in the score. Again using the probability-composition rule, Xenakis calculated (with some approximation [22: 113]) a General Transition Probability Matrix (GTPM, see Table 2). The GTPM ideally captures the complete distribution of probability for the entire work. There one can see that, e.g., screen A has little chances to repeat (2.1%), slightly more chances to be followed by screen C (8.4%), and many more chances to be followed by screen B (35.7%) or F (20.4%). Screen F has strong chances to repeat (20.4%) and to be followed by screen C (30.4%), and little chances to be followed by either B (1.6%) or G (1.8%).

### 3.4 Articulation (a): Binary Logics

Significant internal simmetries are found among and within the 8 screens. Observe screens A, B, C and D. In all of them, pitches are distributed primarily in the extreme registers. In the remaining screens, E, F, G and H, most pitches are mostly in the middle registers. There is a complementarity between the first 4 and the latter 4, and that mirrors the initial arrangment of pitch sets made by Xenakis.

When screens A, B, E and F are used, sounds will be mostly loud (*f* or *ff*), while with the remaining screens they will be mostly *pp*. This contrast, too, stems from the initial, arbitrary arrangement made by Xenakis.

A less dramatic difference is found, instead, concerning the density ranges: screens A, C, E and G have a maximum of 14 events / Δt, while the remaining screens have 16 events / Δt. With such magnitudes, it may be difficult to perceive any substantial difference in density of events, let alone to lean on constrasting areas of density.[22]

As we see, except for density, the musical variables (pitch and dynamics) are governed by an interplay of either complementary or oppository relationships. In principle, we could describe these relationships using Boolean logics operators[23]. Xenakis was thinking more in terms of set thoery, yet we know that set operations are equivalent to Boolean operators[24]. In the making of *Analogique A et B*, he didn't actually utilized the set operations described in the theory, but surely the logics captured by set operations is built in the mechanism as the duality of contrasting and/or

[22] Moreover, as is made clear in [7], the score to *Analogique A* presents variations in density values that can hardly be understood in terms of what the mechanism could produce. In short, it is very hard to even consider that Xenakis really used his compositional mechanism to drive this particular musical variable.

[23] According to Morris Kline, when George Boole (1815-64) introduced his symbolic language capable of expressing the fundamental operations of logical thought, «he had in mind particular applications such as the laws of probability» [14: 201].

[24] Xenakis focused on set operations in the theory of Markovian Stochastic Music [30: 58-59, and 68]. There he mentions the following operations: *intersection*, *union*, *complement*, and *difference*. It should be noted that the intersection, $\cap$, is equivalent with the Boolean operator AND; the union, $\cup$, is equivalent with the Boolean operator OR; and the complement is equivalent to the Boolean NOT. Already in the theory of Free (not Markovian) Stochastic Music, Xenakis had written that «everything in pure determinism… and indeterminism is subject to the fundamental operational laws of logic… disentangled by mathematical thought under the title of general algebra… The most primitive operations… are the union, notated $\cup$, the intersection, notated $\cap$, and negation» [30: 4]. In assigning algebra and set theory a capability to account for the logics of all human constructions [ibid.], Xenakis echoed Georg Cantor, who had claimed, in 1885, that all mathematics could be reduced to set theory [14: 279].

complementing sets of sonic variables (a binary set selection for each variable: $f_0$ or $f_1$, $g_0$ or $f_1$, etc.) and in the resulting relationships between screens. Such a duality is present, too, in Xenakis' decision to dispose of two different TPMs and alternate between them in order to govern the set selection (α or β for pitch, γ or ε for dynamics, etc.).

In short, *there is a binary logics governing the use of probability functions and informing the overall compositional mechanism*.

**3.5 Articulation (b): The Mechanism Applied**

*Analogique A* consists of ten short musical sections. Each section was generated letting the compositional mechanism repeatedly select the screen, starting with a given initial screen. There is little doubt that Xenakis recursively applied the GTPM directly. The operation of the whole mechanism he designed is indeed fully captured in the GTPM, so he didn't need to go through all the steps we have described before illustrating the GTPM. Each time a new screen was picked up by the GTPM, Xenakis calculated the new statistical configuration of values (of pitch, dynamics and density) based on the sets specific to the selected screen (as already mentioned, values must be picked up at random from the sets that are parts of the screen). Values were then mapped onto the available locations in the time-grid. The latter step was implemented apparently without any particular criteria – except surely independent *musical* criteria.

For each section, then, we can thus summarize the whole procedure: (1) application of the GTPM to select a screen, (2) calculation of random numbers and selection of values from sets included in the screen, (3) mapping of values into musical notation. The whole precedure must have been repeated for every next Δt (half-bar, 1.2") in a section. That is, 30 times for sections 1, 2, 3, 5, 6, 7, 8 and 10 (the duration for all these section is 30 x Δt = 15 bars), 35 times for section 4 (whose duration is 35 x Δt = 17.5 bars), and 32 times for section 9 (32 x Δt = 16 bars).

Because each screen provides a different statistical configuration of the three musical variables, and thus determines the content of every next half-bar, what is obtained is a sequence of equally-spaced states in a stochastic process, each state having its own statistical properties. Each next half-bar in *Analogique A* is the next frame in a larger, overriding sequence – much like a theorem included in a ordered set of theorems making up a theory. In that sense, Xenakis' compositional mechanism literally embodies a *theory* (in the scientific sense of a conceptual framework explaining or characterising some given phenomenon in a domain). First and foremost it requires a *method* necessary to link from one separate set element to the next. However, the elements themselves – the contents of each half-bar – are largely pre-determined by the initial arrangement of variables. We must say that the method *is* certainly formalized, but the content of the frames governed by that method is only *loosely* formalized, being mostly determined by decisions Xenakis had made at the beginning, independent of the mechanism' behaviour.

Here we see that the composition of *Analogique A et B* leaves Stochastic Music behind, and moves towards the approach behind Symbolic Music, where a more clear-cut distinction exists between independently-composed musical materials and abstract methods to manipulate those materials. In other words, working on *Analogique A et B* Xenakis eventually set the ground for later efforts Symbolic Music. This could be confirmed by an analysis of the only other work of Markovian Stochastic Music, *Syrmos* (1959, for string orchestra), where apparently the same Markovian chain as in *Analogique A et B* was re-utilized by Xenakis. In *Syrmos* the stochastic process regulates «stochastic transformations of eight basic textures» (composer's introductory lines to the score), the latter consisting in an "ordered set" of somewhat independently shaped materials. In *Nomos Alpha* (1964) the transition to a different approach is complete, and the probabilistic method is replaced by a deterministic one, the famous "rotation of the cube".

In the move from Stochastic to Symbolic Music, Xenakis left behind an attempt to drive the sound material *from within*, using formalized approaches to shape the grain of the sound texture, and re-positioned his compositional approach *above* the sound material. Approaches of the former kind returned later, in the form of more thorough microcompositional methods, i.e. in the form of direct sound synthesis by computer (as in *La Legende d'Er*, UPIC, and GENDYN).[25]

### 3.6 Dialectic (a): The Mechanism Reformed

It should not go unmentioned that most of the ten sections of *Analogique A* actually don't follow from the simple and direct application of the compositional mechanism, but from a distorted application. I am not referring here to many details found in the score that cannot be explained in terms of the compositional mechanism, nor to more qualitative decisions having certainly nothing to do with the overall theoretical frame of Stochastic Music (e.g. the various playing techniques required of the instrumentalists). I am referring, instead, to the fact that Xenakis biased the operation itself of the mechanism. That generated musical results that the mechanism would have never generated otherwise. In actuality, only three Sections (1, 4 and 9) exhibit the evolution of the mechanism in "equilibrium", as Xenakis said (in the sense of *thermodynamic equilibrium*), as they follow the weighted chances comprised in the GTPM. Seven of the ten sections resulted instead from "perturbation" (again Xenakis' term) of the mechanism. That means, most of the piece stems not from the laws captured in the compositional mechanism but from a decision to force the latter to function in ways otherwise impossible. Why Xenakis made such a decision is hard to tell. We may only guess that, after some trials, he realised that the range of musical results would be too poor to articulate a longer musical structure, had he applied the mechanism as such.

To exchange between "equilibrium" and "perturbation" Xenakis devised a higher-level control strategy, that he called *protocol of exchange*. We will not discuss the details here – suffices it to say that, for each next Section, the protocol of exchange provided the mechanism with (1) the initial screen, and (2) the "mode of behaviour" (E = equilibrium, or P = perturbation). In other words, that provided the initial conditions for the stochastic process generating the musical data.

Xenakis was not clear what he precisely did to *perturbate* his mechanism. The analysis data [7] evidence that he must have adopted at least two strategies, one stronger and more destructive of the mechanism's equilibrium, one milder and somehow closer to equilibrium. Sections 2, 6 and 7 stem from strong perturbations of the mechanism, causing it to fix on a single screen (screen A for Section2, screen C for Section 6, screen B for Section 7). Sections 5, 8 and 10 follow from milder perturbations, and appear more varied in their internal articulation, yet not as varied as the Sections where the mechanism is in equilibrium.

Forcing the mechanism «to be displaced towards exceptional regions at odds with its behaviour at equilibrium» [30: 94] certainly represents a subversion, or even a negation of the constructive criteria it was born of. However, in a sense that helped Xenakis to better understand the generative process he had designed, in that he could see to what

[25] On this point, it seems interesting to recall an observation made by Xenakis in 1976: «A large portion of *Musiques Formelles* is actually based on the organization of given sound objects, another portion (the last Chapter) is based instead on a kind of more global perception» [33: 53]. He was referring to the 1971 American edition of the book, hence the «last Chapter» is in fact *New Proposals in Microsound Structure*, which deals with a number of compositional methods for direct sound synthesis by computers (microcomposition). From this observation we see that Xenakis, in the particular circumstance – and contrary to the premises of all Stochastic Music, either Free or Markovian, as found in *Musiques Formelles* – assumed Markovian Stochastic Music to be a very formalistic approach («organization of given sound objects»), not as an empirical approach on the composition of the internal flow of sound.

extent it could remain consistent with itself under arbitrary working conditions. The piece presents the listeners «an entity» (as exposed in the E-Sections) and with a negation or «modification of the entity» (as exposed in the P-Sections) [30: 95]. Listeners can thus make a comparison between *two propositions concerning the same entity*. At stake is, then, the identity of the mechanism. The identity is reinforced as it is deliberately put at risk.

As we listen to *Analogique A* our attention is initially focussed on the internal articulation of each section, but later it can also focus on the dialectic of E-Sections and P-Sections, shifting on larger-scale structural properties. Especially effective, in this sense, is the dramatical opposition between Section 1 and Section 2. The first exhibits the mechanism in perfect thermodynamic equilibrium. The second exhibits the mechanism as it gets strongly "perturbated", so much so that it gets soon stuck on a screen and repeats it over and over again – namely screen A. You can see from the GTPM that screen A is not at all one of the most likely to be selected, and certainly not one on which the mechanism would insist: there is only 2.1% of chances that an instance of screen A is followed by another instance of screen A.

Confronted with the contrast between Equilibrium and Perturbation – between thesis and antithesis – the listener is in the position to compose his or her own synthesis. The composer himself, as a listener, was in a similar position when he first listened to the musical results of the formalized process. Presumably he was himself achieving a decisive moment of synthesis when he opted, at some point in the year 1960, to make *Analogique A* and *Analogique B* overlap. (We shall come back to that point later in the present paper).

**3.7 Dialectic (b): Analog and Digital**

There is a sense of discontinuity, a gap separating successive frames of *Analogique A*. The music unfolds step-wise, pulsating at a rate of 1.2" (a rate of 0.5" in *Analogique B*). Clearly, that is not fast enough to create, for the ear, a sense of gradual and smooth transition, especially then if the items being concatenated include larger shifts in pitch registers and dynamics. What is offered to the listener is a series of snapshots, so to say, capturing equally-spaced moments of a presumably more continuous process flowing in the background. More precisely, what is offered is not a continuous stochastic process, but a sequence of *samples* of that process. Sample Rate = $1/\Delta t = 1/1.2 = 0.83$ Hz (in *Analogique B*, Sample Rate = $1/0.5 = 2$ Hz). That is very evident to the ear, all through the piece, except maybe when some screen gets repeated twice or more (in the P-Sections), making a prolonged sequence of nearly-identical musical frames. And except maybe for sudden general pauses which punctuate some of the ten Sections.[26] In short, we have a *digital* rendering of the *analog* representation of an underlying (but of course only imagined!) stochastic sonic process – the transformations of a *cloud of sounds*. In the theory of Markovian Stochastic Music, Xenakis described the stochastic process in the continuum. But, in the application of the theory, he operated a digital representation of it.

The metaphor of the *nuages de son*, with its atmospheric reference, deserves some attention. It is known that Xenakis' imagination was often inspired by images of physical phenomena. His use of mathematics should be seen primarily as instrumental to a (rough) modeling of physical phenomena.[27] This shows, I think, Xenakis' position with

[26] As discussed in passages of [7], Section 4 (one of the three E-Sections) is punctuated by general pauses which are totally independent of the compositional mechanism. Xenakis created them more by mapping the notes in particular ways across the time-grid, than by levelling-off the density of events prescribed by the mechanism itself. Without such pauses, Section 4 would appear overall very similar with the other E-Sections (1 and 9).

[27] In so doing, all of the Classical and Romantic mathematics Xenakis deployed in the building of the theory of Stochastic Music, ultimately resulted more in a raw, "brute force" approach, in a *finite* system where no such things exist as square roots, fractions, or $\pi$. Here, then, we get closer to a kind of "punk mathematics", with a definition found in [12: 27].

regard to the connection of abstract mathematical thought to empirical domains, a question of relevance in the debates on the foundations of mathematics.[28]

On this point, I will mention two radically opposite views, both of which Xenakis might have been aware of. In 1947, John von Neumann claimed that the most important achievements in pure mathematics came undoubtly from research work in natural sciences, and that the work of mathematicians who keep themselves at a distance from all empirical content is ultimately sterile and meaningless. In sharp contrast, the French Jean Dieudonné – active in the Bourbaki group of mathematicians[29] – claimed in 1964 that none of the major historical achievements in mathematics had anything to do with physics or any other empirical domain (except, he added, for the "theory of distribution", by which Dieudonné presumably meant the theory of probability and other endeavours in statistics).[30]

Von Neumann (who called himself a methematician, but was better known as the prototypical computer engineer) complained that "pure mathematics" is a kind of *art pour l'art* and an end in itself. The paraphernalia of mathematical notations found in Xenakis' writings on Stochastic Music (either Free or Markovian) was clearly not an end in itself. It was meant to create a music as-yet-unheard. Had the case been different, the composer of *Analogique A* would have not subverted nor biased his own mechanism. He would have not made manual adjustments in the score.[31] Finally, he would have not decided to paste together the two pieces, initially separate, *Analogique A* and *Analogique B*. By itself alone, the latter decision calls into question the overall framework of theoretical premises. But in essence all such circumstances – negating the mechanism, adjusting the output data, making one piece out of two – stemmed from qualitative, non-formalized (or at least not-yet-formalized) choices.[32]

Observe, in addition, that since the beginning Xenakis arranged the musical variables in peculiar ways having no special justification on a theoretical level. I refer to (1) a very specific clustering of pitch registers, (2) a reduced range of intensity values, and (3) an utterly arbitrary and pre-determined rhythmic grouping (triplets against quadruplets against quntiplets) serving as a periodic time grid. All that required no special formalism, and can be explained more precisely with an awareness that the particular arrangement of variables would lend itself well to be manipulated by the formalized process.

[28] Incidentally, a book by Maurice Fréchet published in 1955 could have been significant to Xenakis, whose title is *Les mathématiques et le concret* [9]. Two chapters in that book cover very meaningful topics, namely *Sur le calcul de probabilités et ses applications* and *Les mathématiciens et la vie*. Yet, as far as I know, no reference is found to that book in Xenakis' writings, nor the Fréchet book seems to be part of the composer's private library.

[29] The Bourbaki group is included in the bibliographical references of *Musiques Formelles* concerning Set Theory. According to Orcalli [22: 104], the notion itself of "mechanism" or "machine" as in Xenakis must be referred to the Bourbaki. Books from the Bourbaki, and even a book by Dieudonné dealing with "pure mathematics" and with Bourbaki-related issues, were found in Xenakis' private library (Makis Solomos, personal communication).

[30] See [14: 326-7].

[31] See details in [7].

[32] The "protocol of exchanges" consists in the list of the "initial screens" (one of the eight, A to H) and the "modes of behaviour" (equilibrium = E, and either one of two different "perturbations" = $P^0$ and $P^1$) utilized in the manipulation of the mechanism, for each separate section [30: 96 and 105]. If my analysis is correct, it must be considered an *ad-hoc* solution that Xenakis justified only *a-posteriori* on a formal level. The idea is that, based on a need to make the mechanism get more varied musical results than it could, *first* Xenakis manipulated the mechanisms in ways not planned and *then* dubbed *protocol of exchanges* the list of "initial screens" and (importantly) "modes of behaviour" utilized in the manipulation. In short, what he described as an over-ruling device, was probably a kind of posterior rationalization. – Here we have an example, where the experience of what is (musically) missing opens to *not-yet formalized* decisions, and where the latter are finally made an integral element of the formalized framework. *Analogique A* is "formalized music" indeed, but more precisely it is *music in the process of being formalized*.

### 3.8 Position and Negation

Let's finally consider the decision to let *Analogique A* and *Analogique B* overlap between them. Although I can't discuss here how the two were junctioned,[33] but I must at least consider the most evident and manifest phenomenon caused by their overlap, namely the overt timbral difference between the instrumental and the electronic layer. Upon listening, there seems to be no apparent integration between the two, no contact (...keine *Kontakte*...), no smooth transition. Isn't that the unsuccessful montage of mutually irreconcilable entities, of two irreducible sonic worlds? No, it isn't – I would dare saying. The difference is so overt, and the lack of any mimetic relationship and timbral mediation so clear and neat – it's hard to believe that the composer had not heard and thus pondered that carefully.

Not leaning on any mimetic kind of connection, the listener's attention may shift towards deeper structural characteristics, more crucial to this music – towards the dynamic shape of sound clouds developing across different but overalapping temporal levels. The close encounter of the two sonic worlds allows us to make «a sensorial and structural comparison» of two non-identical manifestations of the same compositional process [32: 31]. The *same* is presented as *different*, projected on different time-scales. Another analogy is established, and presented to the listener, the one between the macro-scale structure (as captured in the instrumental score) and the micro-scale structure (as captured in the texture of sonic atoms). In other words, by preserving the surface difference, Xenakis pointed to the manifestation of a more profound identity.

We have an echo, here, of the decision Xenakis had already made for himself, the decision that the mechanism should be "perturbated". In that occasion, too, was the goal to let an apparent contrast (of E-Sections and P-Sections) better reveal a more profound identity – the identity of the stochastic mechanism, with its consistent logic.

In both cases the composer *listened to* the results achieved, *evaluated* their coherence, and made decisions in order to reinforce or support that coherence. The rationality proper to evaluations and decisions were most probably of a kind other than that captured in the formalized process, perhaps originating from non-formalizable, more qualitative criteria. Yet, evaluations and decisions dealt with and acted upon fact and data first brought forth by the rationality captured in and manifested by the formalized process itself. In a way not at all metaphorical, the mechanism here represents the composer's *alter ego*: it sets the ground and the boundaries for more intuitive choices to be made.

«The aim of mathematical rigour is only to sanction
and legitimate the conquests of intuition» (Jacques Hadamard)

## 4. CONCLUSIONS (LESSONS TO LEARN)

Hadamard's statement (quoted in [14: 347]) is certainly pertinent in this context, and it may help us frame, in conclusion, the relationship of an artist like Xenakis to mathematical thinking. Yet, if my analysis is correct, that statement must be turned round: in Xenakis, the conquests of intuition sanction and legitimate the premises of mathematical rigour. *Analogique A et B* presents the listener with a problematic encounter or clash between efforts to formalize the process of music composing, and a cluster of direct, empirical choices not integral to the initial rational efforts. What it really offers to the our ears and mind is an awareness that

(1) *intuitive decisions are legitimate and effective if and when a deep awareness of the creative process has been developed and eventually formalized (or otherwise rationalized)*

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[33] See analytical details in [7].

and

(2) *efforts in formalization (or other form of rationalization) of knowledge are legitimate and effective if and when one is confident that intuition will complete the job in case they might reveal insufficient.*

Because the latter statement is always the case, and because it needs the former statement to be correct for the latter to be the case, the former statement is also correct.

In listening to *Analogique A et B* we are in the presence of music that stimulates a listening experience *analog to the way in which the composer listened to his own mechanism and creatively reacted to it, negating it (hence negating himself) in order to better affirm himself (and hence negating it)*. That is always the case: we must, to some extent, negate ourselves and welcome the non-identical (which could also be within us, not foreign to us) in order to reinforce our identity. Affirmation of identity through exclusion of the non-identical is excluded.

The lesson we can (and I must) learn, is: the more one rationalizes one's own composing, and the more the quality and meaning of the music will also result from non-formalizable choices reflecting not-yet-pondered attitudes and idiosyncracies. On the other hand, the more one leans (or pretends she or he can lean) on purely aesthetical, intuitive, ineffable choices, the more the quality and meaning of the music will result from pre-determined and formalistic decisions and choices – and, even worse, not one's own personal decisions and choices… The former chance allows one to appropriate the music (or any action one takes while living in this world), albeit only to a relative degree of course, in any case with no prior warranty of succesfull appropriation. The latter chance accomodates on the good premises for the music (or any action one takes) to be alienated – probably not entirely and only to some degree, but with the warranty and certainty that nobody will be aware of it and will complain.

It is significant that such a lesson comes from a time when, based on Information Theory and Cybernetics, early but decisive developments of communication technologies were taking place, that later have become pervasive: in the broader social context, technology (as the medium of action reflecting the knowledge it makes applicable) was then turning from a *tool* to an *environment* – it was in the process of becoming the over-technologized world we live in, today, our new *Lebenswelt*. And it is significant that the lesson comes from a man who, based on a strong need for self-determination and self-awareness – hence, a need for a deep awareness of the the historical context of his time – preferred to be an active part of that changing world, rather than a passive part, shaped his working environment and thus appropriated the music borne of it. *Analogique A et B* is a small but precious testimony of hidden but soon to reveal largely relevant trends taking place at the time of its composing. Art not only *mirrors* but *participates in the making* of the social and cultural scenario of its time.

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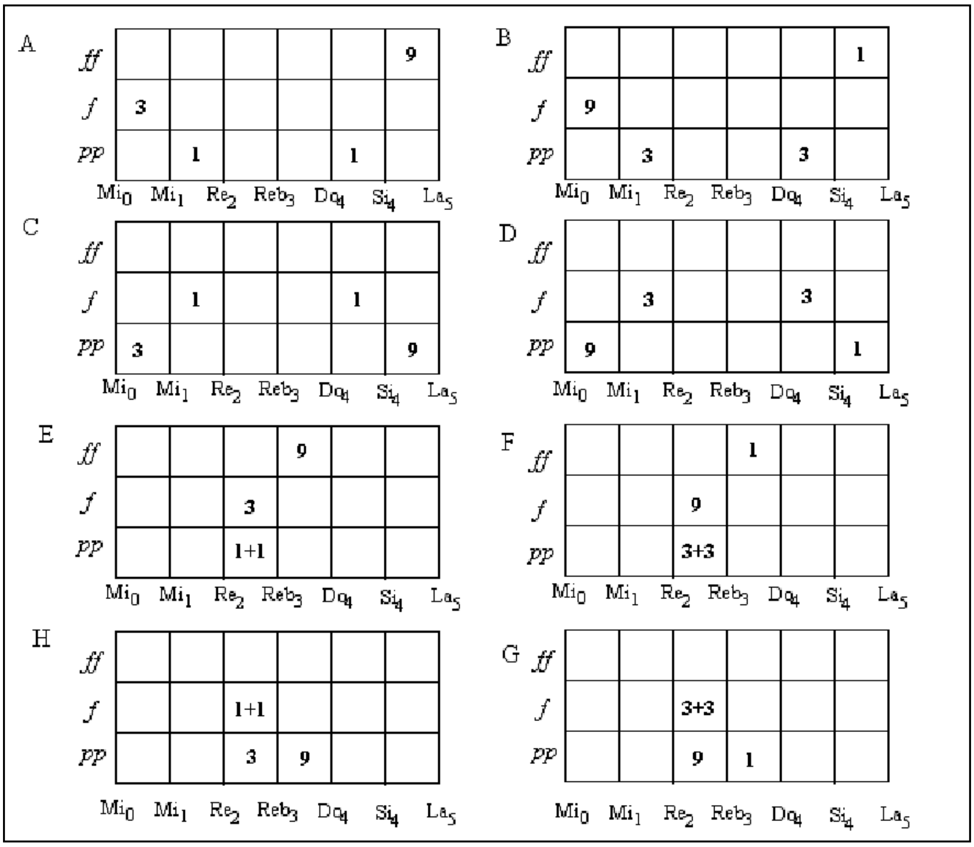


**Table 1**

The eight "screens" determining the statistical profiles for the musical contents in each half-bar in the score to *Analogique A*.

**GENERAL TRANSITION PROBABILITY MATRIX**

| | A($f_0 g_0 d_0$) | B($f_0 g_0 d_1$) | C($f_0 g_1 d_0$) | D($f_0 g_1 d_1$) | E($f_1 g_0 d_0$) | F($f_1 g_0 d_1$) | G($f_1 g_1 d_0$) | H($f_1 g_1 d_1$) |
|---|---|---|---|---|---|---|---|---|
| A($f_0 g_0 d_0$) | 0.021 | 0.357 | 0.084 | 0.189 | 0.165 | 0.204 | 0.408 | 0.096 |
| B($f_0 g_0 d_1$) | 0.084 | 0.089 | 0.076 | 0.126 | 0.15 | 0.136 | 0.072 | 0.144 |
| C($f_0 g_1 d_0$) | 0.084 | 0.323 | 0.021 | 0.126 | 0.15 | 0.036 | 0.272 | 0.144 |
| D($f_0 g_1 d_1$) | 0.336 | 0.081 | 0.019 | 0.084 | 0.135 | 0.024 | 0.048 | 0.216 |
| E($f_1 g_0 d_0$) | 0.019 | 0.063 | 0.336 | 0.171 | 0.11 | 0.306 | 0.102 | 0.064 |
| F($f_1 g_0 d_1$) | 0.076 | 0.016 | 0.304 | 0.114 | 0.1 | 0.204 | 0.018 | 0.096 |
| G($f_1 g_1 d_0$) | 0.076 | 0.057 | 0.084 | 0.114 | 0.1 | 0.054 | 0.068 | 0.096 |
| H($f_1 g_1 d_1$) | 0.304 | 0.014 | 0.076 | 0.076 | 0.09 | 0.036 | 0.012 | 0.144 |

**Table 2**

The General Transition Probability Matrix used by Xenakis for the implementation of his "mechanism" (Markovian chain). The application of the GTPM, determined the sequence of "screens" assigned to each next half-bar in the score to *Analogique A*.